Journal of Applied Mechanics and Technical Physics, Vol. 44, No. 6, pp. 872–879, 2003

HYDRODYNAMIC MODEL FOR THE EVOLUTION OF AN INVADED ZONE IN BOREHOLE DRILLING

A. A. Kashevarov,[1] I. N. Yeltsov,[2] and M. I. Épov[2]

UDC 532.546:550.820.7

The evolution of an invaded zone during borehole drilling in water- and oil-saturated sand formations is studied by mathematical modeling of hydrodynamic processes in porous media filled with a two-component fluid. The use of hydrodynamic modeling to interpret high-frequency electromagnetic logs makes it possible to construct consistent geoelectric and hydrodynamic models for formations with different fluid saturations. The results obtained are in good agreement with geological and geophysical data.

Key words: *mathematical modeling, invaded zone, formation, oil saturation, concentration, resistivity, borehole, logging.*

Introduction. Under the action of a high pressure drop produced during borehole drilling, the drilling mud invades an oil-saturated formation and displaces the pore fluids — water and oil. The filtration process is rapidly decelerated by the growth of a mud cake on the borehole wall, and after cessation of drilling, the borehole and formation pressure are equalized. Further evolution is driven by slow processes — differentiation of fluids of different densities and multiphase flow under the action of capillary forces. The salt contents in the drilling mud and pore water are usually different. This difference, combined with the nonuniform distribution of the oil phase in the borehole region, causes a significant change in the electrical properties of the formation. Hydrodynamic modeling allows one to reconstruct the concentration and water-saturation distributions from the invaded-zone resistivity and to determine the hydrophysical properties of the formation using a generalized Archi formula.

Hydrodynamic Model. In straight-hole drilling, the main factors determining processes in the near-borehole region are mud circulation, the growth and erosion of a mud cake, fluid (water and oil) filtration, and salt transport. We consider the initial stage of evolution of an invaded zone, where the vertical drift due to the different fluid densities and the effect of capillary forces are insignificant.

In cylindrical coordinates, two-phase filtration is described by the Buckley–Leverette equations subject to the additional condition on the sum of the water and oil saturations $S_{\rm w} + S_{\rm oil} = 1$ [1]. For a two-dimensional region $\Omega = \{r_{\rm b} < r < L,\ 0 < z < L_z\}$ there is a system of two transport equations for the mobile phases:

— for water saturation $S_{\rm w}$,

$$\frac{\partial}{\partial t}(rmS_{\rm w}) = \frac{\partial}{\partial r}\Big(rk_{\rm w}\,\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_{\rm w}\,\frac{\partial h}{\partial z}\Big), \qquad h = \frac{p}{\gamma} + z, \quad (r,z) \in \Omega; \tag{1}$$

— for oil saturation $S_{\rm oil}$,

$$\frac{\partial}{\partial t}(rmS_{\rm oil}) = \frac{\partial}{\partial r}\Big(rk_{\rm oil}\,\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(rk_{\rm oil}\,\frac{\partial h}{\partial z}\Big), \qquad m = m_0(z) + \delta(p - p_f(z)). \tag{2}$$

Here the z axis of the coordinate system is directed upward and coincides with the axis of a borehole of radius $r_{\rm b}$, p is the pressure, δ is the elasticity coefficient, h is the hydrodynamic head, γ is the pore fluid density, m is the formation porosity, $k_{\rm w} = k_0 S^{n_1}$ and $k_{\rm oil} = \mu_0 k_0 S^{n_2}$ are functions that define the phase permeabilities of the water and oil phases, respectively; and $\mu_0 = \mu_{\rm w}/\mu_{\rm oil}$ is the ratio of the water and oil viscosities.

[1]Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. [2]Institute of Geophysics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 44, No. 6, pp. 148–157, November–December, 2003. Original article submitted March 26, 2003; revision submitted May 13, 2003.

 0021-8944/03/4406-0872 $25.00 © 2003 Plenum Publishing Corporation

Taking into account the residual water saturation ($s^1_{\min}$) and oil saturation ($s^2_{\min}$), in Eqs. (1) and (2) we have $S_{\rm w} = (s_{\rm w} - s^1_{\min})/(1 - s^1_{\min} - s^2_{\min})$ and $S_{\rm oil} = (s_{\rm oil} - s^2_{\min})/(1 - s^1_{\min} - s^2_{\min})$, where $s_{\rm w}$ and $s_{\rm oil}$ are the saturations of the corresponding phases [2].

We specify boundary and initial conditions assuming that at the initial time, the head is equal to the formation head h_f:

$$h\Big|_{r=L} = h_f, \qquad \frac{\partial h}{\partial z}\Big|_{z=0,\, z=L_z} = 0, \qquad h\Big|_{t=0} = h_f.$$

On the left boundary ($r = r_{\rm b}$), the condition is specified with allowance for the position of the bottom hole [$z = l_{\rm b}(t)$]:

$$-q = (k_{\rm w} + k_{\rm oil})\frac{\partial h}{\partial r}\Big|_{r=r_{\rm b}} = \beta(p\Big|_{r=r_{\rm b}} - p_{\rm b}), \qquad l_{\rm b} \leqslant z \leqslant L_z, \qquad \frac{\partial h}{\partial r}\Big|_{r=r_{\rm b}} = 0, \qquad 0 \leqslant z < l_{\rm b}. \tag{3}$$

If the flow is directed from the boundary into the region Ω, it is necessary to specify the water saturation on the corresponding segment of the boundary:

$$S_{\rm w}\Big|_{r=r_{\rm b}} = 1, \qquad S_{\rm w}\Big|_{r=L} = S_f, \qquad S_{\rm w}\Big|_{t=0} = S_f.$$

For the borehole, the drilling mud flow rate $Q(t)$ and the drilling rate $V(t)$ are specified. We assume that the drilling mud is incompressible and its losses by filtration into the formation can be ignored. In this case, the pressure distribution in the circulation system is calculated from the Darcy–Weissbach formula using the well-known procedure of [3, 4]. On a segment of length l, the hydrodynamic losses of the pressure are equal to

$$\Delta P = \lambda \gamma_{\rm b} l v^2/(8gR_{\rm h})$$

($v = Q/\omega$ is the mean flow rate, ω is the sectional area, $R_{\rm h}$ is the hydraulic radius, λ is the hydraulic resistance coefficient, and $\gamma_{\rm b}$ is the mud density).

The hydraulic resistance coefficient in different borehole sections depends on the type of flow. Turbulent flow is formed under the following constraint on the Reynolds number $\mathrm{Re} = vd\rho/\mu$:

$$\mathrm{Re} > \mathrm{Re}^*\,.$$

Here $\mathrm{Re}^* = 2100 + 7.3\,\mathrm{He}^{0.58}$, where $\mathrm{He} = \rho d^2\tau/\eta^2$ is the Hedstrom number, d is the clearance diameter, τ is the dynamic shear stress, η is the plastic (dynamic) viscosity, and ρ is the density. In this case, the value of λ in the hole annulus (with a clearance diameter d) can be calculated from the formula $\lambda = \lambda_0(k/d + b/\mathrm{Re})^{0.25}$ [4]. The roughness coefficient k is different for the cased and uncased borehole sections.

The surface pressure is equal to atmospheric pressure, and, hence, calculating ΔP in different sections, one can calculate the pressure distribution over the entire borehole and its time variation during drilling of the formation.

Considering the drilling mud flow to the bottom hole and taking into account the pressure drop in the drilling tool, one can calculate the hydrodynamic pressure losses in the entire circulation system. Measuring the drilling mud flow rate and head (for different drilling depths), one can determine the parameters of the model and calculate the excess of the borehole pressure over the formation pressure.

Salt transport is modeled by the transport equation for a conservative impurity [2]. The effect of hydrodynamic dispersion is considered negligible. The relative concentration C of the salts transported by the water phase obeys the equation

$$\begin{gathered} \frac{\partial}{\partial t}\,(rmS_{\rm w}C) + \frac{\partial}{\partial r}\,(v_rC) + \frac{\partial}{\partial z}\,(v_zC) = 0, \\ v_r = -rk_{\rm w}\,\frac{\partial h}{\partial r}, \qquad v_z = -rk_{\rm w}\,\frac{\partial h}{\partial z}, \qquad C\Big|_{t=0} = C_f. \end{gathered} \tag{4}$$

Boundary conditions for the salt-transport equation are specified only in those sections in which the fluid with variable concentration enters the formation through the boundary of the modeled region. In this case, the concentration is set equal to the formation water concentration ($C\Big|_{r=L} = C_f$) or the mud filtrate concentration ($C\Big|_{r=r_{\rm b}} = C_{\rm mud}$).

The rate of mud cake growth is proportional to the drilling mud losses into the formation. Transport of clay particles into the formation and porosity variation in the near-borehole region are ignored. The thickness of the mud cake $d(z,t)$ is included in the parameter $\beta = (\beta_0^{-1} + d/k_{\rm clay})^{-1}$, which determines water exchange between the borehole and the formation and allows for the filtration resistance at the initial moment of drilling-in β_0^{-1} and the

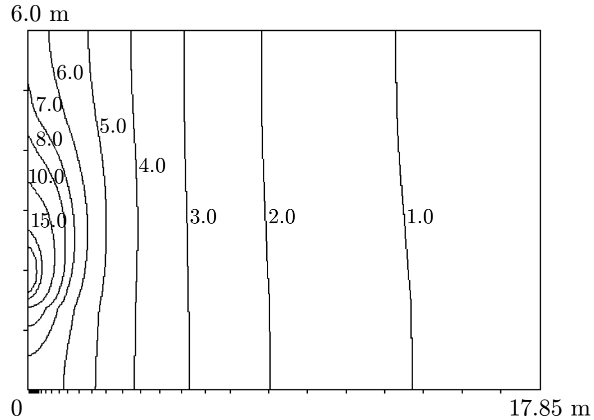

Fig. 1. Isolines of the excess of the formation hydrodynamic head.

additional resistance due to the build-up of the mud cake (d/k_{clay}). The rate of mud cake growth is proportional to the rate of filtration from the borehole into the formation and is defined by the equation

$$d'_t = -\alpha(1 - d/d_{\max})^n q \qquad (n \geqslant 0),$$

where k_{clay} is the filtration coefficient of the mud cake and $d_{\max}$ is the maximum cake thickness. The value of α depends on the mud-cake porosity, the fraction of clay particles in the drilling mud, and the other parameters that determine conditions for mud-cake growth.

Numerical Modeling. In numerical implementation, instead of the oil saturation equation, we used the equation for the total flow rate [1, 2] obtained by summation of Eqs.(1) and (2):

$$\frac{\partial}{\partial t}(rm) = \frac{\partial}{\partial r}\Big(r(k_{\mathrm{w}} + k_{\mathrm{oil}})\frac{\partial h}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(r(k_{\mathrm{w}} + k_{\mathrm{oil}})\frac{\partial h}{\partial z}\Big). \tag{5}$$

The calculation begins at the moment of initial bit penetration into the formation overlain and underlain by impermeable clay beds. In this case, boundary condition (3) on the borehole wall for the borehole h_{b} and formation h pressure heads are written as

$$-q = (k_{\mathrm{w}} + k_{\mathrm{oil}})\frac{\partial h}{\partial r}\Big|_{r=r_{\mathrm{b}}} = \beta(h\Big|_{r=r_{\mathrm{b}}} - h_{\mathrm{b}}), \qquad l_{\mathrm{b}}(t) \leqslant z \leqslant L_z.$$

At the formation roof, an additional pressure is specified that allows for the difference in density between the drilling mud and the pore fluid.

The numerical calculations were performed using implicit finite-difference schemes and an iterative method of variable directions [5]. For the transport equations, a countercurrent approximation was used [6].

The model problem of the displacement of formation fluids from the borehole by the mud filtrate was solved for the following data (example 1). The modeling region ($L = 30$ m and $L_z = 6$ m), representing a separate formation, is partitioned by a difference grid with 48 and 41 nodes along the radius and the vertical coordinate, respectively. At the formation roof and bottom, the head is equal to zero; and on the right boundary ($r = L$), a constant pressure equal to the formation pressure is set. The grid sizes are variable along the radius (minimal value near the borehole 0.025 m) and constant on the vertical z coordinate. The process parameters are as follows: drilling rate 200 m/days, drilling-tool diameter 0.170 m and its length 30 m, and borehole wall 0.216 m. The depth of the oil-saturated formation is 2000 m. The formation 6 m thick is divided into four layers, each 1.5 m thick, with identical hydrophysical properties ($k = 0.15$ m/day, $m = 0.20$, and $\delta = 10^5$) but with different oil saturation values: $S_{\mathrm{oil}} = 0.9$ (from the formation roof to a depth of 1.5 m), 0.75 (from 1.5 to 3 m), 0.45 (from 3 to 4.5 m), and 0 (from a depth of 4.5 m to the formation base). In the formulas of phase permeabilities, the exponents are $n_1 = n_2 = 2.5$ and the ratio of the viscosities of the formation water and oil is $\mu_0 = 0.3$. The salt contents are $C_f = 19.0$ g/liter in formation water and $C_{\mathrm{mud}} = 0.9$ g/liter in the drilling mud, the filtration coefficient of the mud cake is $k_{\mathrm{clay}} = 8 \cdot 10^{-6}$ m/day, $\alpha = 0.06$, and $n = 0$.

Figure 1 gives isolines of the excess of the head h over the initial value h_f after passage of the drilling bit through the formation ($t = 0.02$ days). Figure 2 gives the water-saturation and concentration distributions in the

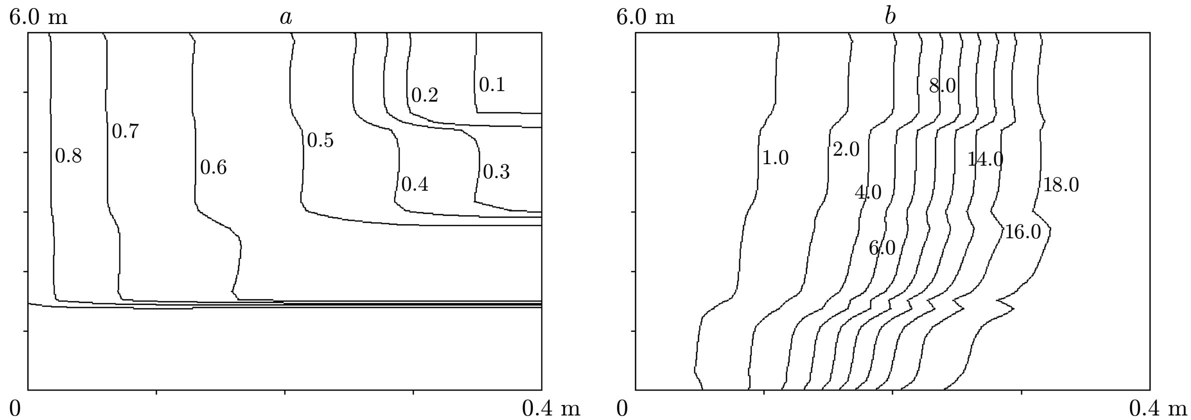

Fig. 2. Isolines of water saturation (a) and salt content (b) in the invaded zone.

near-borehole region at the moment of cessation of drilling ($t = 0.5$ days). The dimensions of fragments of the region and the difference grid are indicated in Figs. 1–3.

For the time interval considered, the width of the invaded zone determined from the concentration variation does not exceed 0.4 m. It is maximal in the oil-saturated portion of the formation and decreases sharply in the water-saturated portion. However, the specific volume Q_0 of the mud filtrate invading the formation varies insignificantly. For the middle parts of the layers beginning with the upper layer, the values $Q_0/\pi = 0.0066$ 0.0063, 0.0065, and 0.0069 m^2 are obtained. For the upper two layers, the radial resistivity profile has a local minimum. This change in resistivity is characteristic of only productive reservoirs and can be detected in high-frequency electromagnetic logging (HFEL).

We consider the main physical features of the mud filtrate inflow into the reservoir and dynamics of an invaded zone. In this process, three stages can be distinguished. At the first stage, because of hindered mud circulation near the bottom hole there is a zone of elevated pressure with a high head gradient not only along the radial coordinate but also along the borehole axis. At the second stage, an important factor in the evolution of invaded zones is cake growth on the borehole wall, which significantly hinders water-exchange processes between the borehole and the formation. At initial times, there is displacement of formation water and oil from the bottom hole and the fluid flow has a spatial nature. After the drilling bit penetrates the formation, the displacement process is primarily due to an excess of the constant hydrodynamic borehole head over the formation head and has a predominantly radial nature. The third stage of the evolution of an invaded zone begins after termination of the drilling process. Mud circulation ceases, and a hydrostatic pressure distribution is established in the borehole. As at the second stage, this leads to radial fluid displacement from the borehole zone. The formation and borehole pressures are rapidly equalized. At the third stage, an increase in the radial depth of the invaded zone is due to a long time of action of relatively low pressure drops in the borehole and formation.

Resistivity of the Invaded Zone and Solution of the Inverse Problem. The invaded-zone resistivity profile results from interaction of concentration and water saturation distributions. It can be calculated using the generalized Archi formula [7]:

$$R = A\,(C + C_0)^{-p}(S + S_0)^{-q}(m + m_0)^{-q} f(T). \tag{6}$$

For specified values of oil saturation and concentrations of drilling mud and formation water, formula (6) gives a fairly accurate approximation of the resistivity determined from well logging sounding data. It is necessary to use resistivities at points nearest to the borehole, in which formation water is completely displaced, as a rule, and at points outside the invaded zone at large distances from the borehole.

The fluid distributions obtained by numerical modeling in the previous section (see example 1) using formula (6) can be transformed to a spatial resistivity profile around the borehole. Figure 3 shows resistivity profiles for the middle parts of the layers plotted for the following parameter values in the Archi formula: $A = 1.4$, $p = 1$, and $q = 2$. This profiles can also be derived from electromagnetic sounding data. Thus, a model relationship is established between the results of inversion of electromagnetic logs and fluid distribution. The hydrodynamic modeling results are in good agreement with logging data [8].

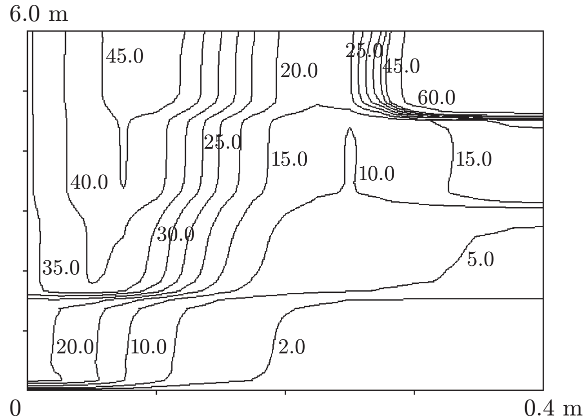

Fig. 3. Resistivity isolines in the invaded zone.

The inverse problem was solved using HFEL data. The device for HFEL (Russian abbreviation VÉMKZ) incorporates nine three-coil arrays. Signals oscillate in the range 0.875–14.0 MHz. The array spacing is 0.5–2.0 m. Phase differences are measured between closely spaced receiver coils [9].

Radial resistivity profiles were obtained from the results of log inversion with the MFS VIKIZ-98 software [10]. Direct two-dimensional hydrodynamic modeling was used to approximate the distribution functions of the concentration $C = \varphi_1(r, d_1)$ in the interval $l_1 < r < l_2$ and water saturation $S_{\rm w} = \varphi_2(r, d_2)$ in the invaded zone $r_{\rm b} < r < l_2$ (l_1 and l_2 are geometrical parameters of the problem that determine the spatial distributions of concentration and water saturation). The parameters d_1 and d_2 allow for the physical processes involved in the drilling of a particular borehole (smearing of the fronts, oil displacement) and determine the nature of the concentration and water saturation distribution curves. The functions φ_i are subjected to the additional conditions

$$\varphi_1(r, d_1) = C_{\rm mud}, \quad r \leqslant l_1, \qquad \varphi_1(l_2, d_1) = C_f, \qquad \varphi_2(r_{\rm b}, d_2) = 1, \qquad \varphi_2(l_2, d_2) = S_f.$$

As the approximating function for the water saturation, we used an analytical solution of the axisymmetric problem ($m = \text{const}$) that takes into account the smearing of the front in the interval $(l_2 - d_2/2, l_2 + d_2/2)$, and the concentration was approximated by the dependence

$$C(r) = C^0 F(r) + C_{\rm mud}, \qquad C^0 = \frac{C_f - C_{\rm mud}}{F(l_2)}, \qquad F(r) = \frac{(r - l_1)^{3+d_1}}{3 + d_1} - (l_2 - l_1)\,\frac{(r - l_1)^{2+d_1}}{2 + d_1}.$$

The invaded zone was divided into intervals (r_{i-1}, r_i), in which the resistivity $R(r)$ was evaluated using the Archi formula. The geometrical parameters l_1 and l_2 were determined by solving the problem of minimization of the functional derived to reconstruct measured values of the total conductivity σ_i:

$$J_\sigma = \sum_{i=1}^{N} \lambda_i \Big((r_i - r_{i-1}) \Big/ \int\limits_{r_{i-1}}^{r_i} R(r)\, dr - \sigma_i \Big)^2,$$

where N is the number of intervals and λ_i are weight coefficients.

The resistivity of the undisturbed formation $R_f = 1/\sigma_{\rm oil}$ behind the invaded zone ($r > l_2$) was used to normalize the petrophysical properties of the formation. After elucidation of features of the filtrate invasion, the additional parameters d_1 and d_2 allow one to match the inversion results with the characteristics of neighboring formations. If the borehole drilling conditions are unchanged, the parameters d_1 and d_2 are nearly identical for formations with close petrophysical properties.

A rough estimate of the porosity was obtained using two values of resistivity corresponding to measurements near the borehole (in the zone of complete displacement of formation water) and in the undisturbed formation. Refinement of the porosity and determination of the filtration coefficient were performed using balance relations for the volumes of the mud filtrate and the oil phase displaced from the invaded zone. The greatest difficulty in solving the inverse problem lies in the fact that the value of the coefficient A in formula (6) is unknown. If the coefficient is specified, then the formation porosity can be determined for a known ratio of formation-water and mud filtrate concentrations. Otherwise, one can obtain only the value of a certain generalized parameter $M = A(m + m_0)^{-q}$.

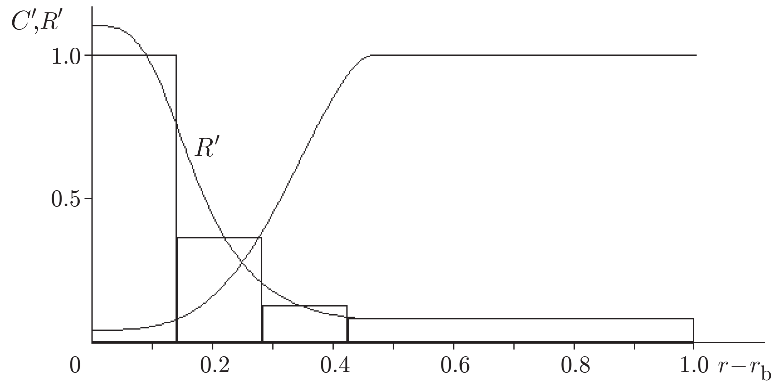

Fig. 4. Radial profiles of concentration and resistivity.

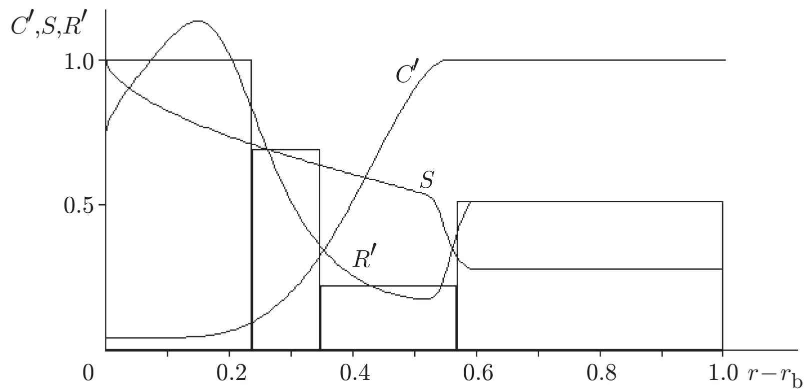

Fig. 5. Radial profiles of concentration, water saturation, and resistivity.

After solution of the inverse problem, hydrodynamic modeling is performed on the basis of a two-dimensional model for the dynamics of an invaded zone with fitted parameters and the results are compared to electromagnetic logging data.

We give results of interpretation of field data obtained in an exploratory borehole in West Siberia (Figs. 4 and 5). The borehole was logged in ten days after drilling for portions of water-saturated and productive formations (examples 2 and 3). The salt content in these formations was set equal to $C_f = 19$ g/liter and in the drilling mud, it was $C_0 = 0.9$ g/liter. In the examples considered, the parameters $A = 1.4$ and $m_0 = 0$ are constant.

The parameters of the geoelectric model obtained from the results of log inversion (resistivities ρ_i [$\Omega \cdot$ m] and thicknesses of radial zones around the borehole Δr_i [m]) have the following values: $\rho_1 = 18.1$, $\Delta r_1 = 0.142$, $\rho_2 = 6.65$, $\Delta r_2 = 0.142$, $\rho_3 = 2.40$, $\Delta r_3 = 0.142$, $\rho_4 = 1.54$, $\Delta r_4 = 1.0$ (example 2); $\rho_1 = 46.18$, $\Delta r_1 = 0.265$, $\rho_2 = 18.46$, $\Delta r_2 = 0.176$, $\rho_3 = 9.80$, $\Delta r_3 = 0.32$, $\rho_4 = 23.22$, and $\Delta r_4 = 1.0$ (example 3). Figures 4 and 5 gives radial profiles of the relative concentration ($C' = C/C_f$), relative resistivity ($R' = R/\rho_1$), and water saturation obtained by solving the inverse problem (examples 2 and 3, respectively). Measured dependences of the resistivity ρ_i on the radius are shown by rectangles.

Figure 4 gives results for a water-saturated formation (depth 1035 m) with a resistivity $\rho_f \approx 1.54\ \Omega \cdot$ m. The radial depth of the invaded zone is approximately 0.4 m. In the most severely flushed zone, the resistivity is approximately 18 $\Omega \cdot$ m; with distance from the borehole, it rapidly decreases. Lithologically, the formation is made up of moderately cemented sandstones; the cementation factor is $q \approx 2.0$. The porosity derived from interpretation is 0.215 ($C_0 = 0.7$).

Figure 5 shows the inversion results for an oil-saturated ($S_{\rm w} \approx 0.72$) layer at a depth of 2300–2310 m. The maximum depth of drilling mud invasion is approximately 0.57 m. In the flushed zone near the borehole, the resistivity is 23.6 $\Omega \cdot$ m; next follows a low-resistivity zone. Figure 5 gives profiles of the concentration C and water saturation S obtained by inversion and the resistivities R calculated from the Archi formula. The porosity determined by inversion is 0.22 ($C_0 = 0.54$ and $S_0 = 0.07$).

Additional information on the physical properties of fluids and the drilling conditions reduces the parametric uncertainty of the hydrodynamic inversion.

Hydrodynamic Analysis Based on Balance Relations. We assume that in the near-borehole region the drilling mud flows into the formation. After cessation of drilling at time T, the formation pressure is equalized and the depth of the invaded zone does not exceed L. The spatial nature of the fluid flow leads to spread of the concentration and water-saturation fronts, which can be taken into account by introducing a dummy hydrodynamic dispersion. In this case, the balance mass-transfer characteristics can be estimated using one-dimensional equations.

The estimates given in the present section are approximate and are valid only for the middle parts of fairly thick layers (more than 1 m), for which water exchange between layers can be neglected. The relations obtained for balance characteristics allow one to estimate the credibility value of hydrodynamic inversion and to check the accuracy of inversion of electromagnetic logging data.

We consider Eqs. (1), (4), and (5) for the one-dimensional axisymmetric case. Integrating them over the radius r and time t, we obtain the balance relations

$$I_1 \equiv \pi \int\limits_{r_\mathrm{b}}^{L} rm\bar{s}\,dr\Big|_T = -\pi \int\limits_0^T v_\mathrm{w}\,dt\Big|_{r=L} + \pi \int\limits_0^T v_\mathrm{w}\,dt\Big|_{r=r_\mathrm{b}} = -Q_\mathrm{w}(L) + Q_\mathrm{w}(r_\mathrm{b}), \quad \bar{s} = S_\mathrm{w} - S_f; \tag{7}$$

$$I_2 \equiv \pi \int\limits_{r_\mathrm{b}}^{L} rm\bar{c}S_\mathrm{w}\,dr\Big|_T = \pi \int\limits_0^T v_\mathrm{w}\,dt\Big|_{r=r_\mathrm{b}} = Q_\mathrm{w}(r_\mathrm{b}), \qquad \bar{c} = \frac{C - C_f}{C_\mathrm{mud} - C_f},$$

$$Q_0(L) = \pi \int\limits_0^T v\,dt\Big|_{r=L} = \pi \int\limits_0^T v\,dt\Big|_{r=r_\mathrm{b}} = Q_0(r_\mathrm{b}) = Q_\mathrm{w}(r_\mathrm{b}), \qquad v = v_\mathrm{w} + v_\mathrm{oil},$$

where v_w and v_oil are functions that define losses of the water and oil phases, respectively, and Q_w is the water flow rate integrated over time.

Assuming that on the right boundary ($r = L$), the phase losses are proportional to the phase mobilities, we obtain

$$Q_\mathrm{w}(L) = \frac{k_\mathrm{w}(S_f)}{k_\mathrm{w}(S_f) + k_\mathrm{oil}(1 - S_f)}\,Q_0(L) = \chi(S_f)Q_0(L) = \chi(S_f)Q_\mathrm{w}(r_\mathrm{b}).$$

In view of the last equality in (7), we obtain the relation

$$I_1 = (1 - \chi(S_f))Q_\mathrm{w}(r_\mathrm{b}) = (1 - \chi(S_f))I_2. \tag{8}$$

Similarly, for a water-saturated formation ($S_\mathrm{w} = 1$), we have

$$I_3 \equiv \pi \int\limits_{r_\mathrm{b}}^{L} rm\bar{c}\,dr\Big|_T = \pi \int\limits_0^T v\,dt\Big|_{r=r_\mathrm{b}} = Q_0(r_\mathrm{b}).$$

Integrating the equation for the total loss (5) over time, we obtain

$$(rkP_r)_r = 0, \qquad P = \int\limits_0^T h\,dt.$$

Therefore, the solution is written as

$$P(r) = P(D) + k^{-1}Q_0 \ln{(D/r)},$$

where $D > L$ is the radius of influence of the borehole.

We consider the condition of the third kind on the left boundary ($r = r_\mathrm{b}$):

$$-r_\mathrm{b}q(t) = r_\mathrm{b}kp_r\Big|_{r=r_\mathrm{b}} = r_\mathrm{b}(k_0/d(t))(h - h_\mathrm{b}(t))\Big|_{rb}. \tag{9}$$

From this, in view of the linear law of cake formation ($d_t = \alpha q$, $n = 0$), it follows that $d(t) = \alpha \int\limits_0^t q\,dt$. Then, from (9) for the specific volume of the filtrate $Q_0(t) = \pi r_\mathrm{b} \int\limits_0^t q\,dt$, we have

$$-(Q_0^2)_t/2 = \pi^2(r_\mathrm{b}k_0/\alpha)(h - h_\mathrm{b}(t))\Big|_{r_c}.$$

Integrating the last equality over time, we obtain

$$-0.5Q_0^2 = \pi^2\,\frac{r_\mathrm{b}k_0}{a}\,(P - F)\Big|_{r_c}, \qquad F(t) = \int\limits_0^t h_\mathrm{b}\,dt. \tag{10}$$

Thus, Eq. (10) relates the specific volume of the filtrate Q_0 to the integrated borehole pressure via the parameter F, which depends on the drilling practices and the borehole design.

The hydrodynamic modeling results show that the volume of the mud filtrate Q_0 invading the formation is determined primarily by the drilling conditions and depends weakly on the filtration coefficient. For examples 2 and 3, the values of 0.0183π and 0.0192π, respectively, were obtained. If the formation void space is partly occupied by oil, the depth of the invaded zone considerably increases compared to a water-saturated formation. For an oil-saturated formation (see example 3), the balance characteristic of the water phase is $I_1 = 0.0173\pi$. In this case, $Q_0 = I_2$, and, hence, from (8) it follows that $\chi(S_f) = 0.0862$, which is in good agreement with the employed dependences of phase permeabilities in hydrodynamic inversion $\chi(S_f) = 0.0945$ for $n_1 = n_2 = 2.5$, $S_f = 0.28$, $\mu_0 = 0.4$.

Conclusions. Based on an electrohydrodynamic model, a system of inversion for high-frequency electromagnetic logging data was developed that allows the construction of consistent geoelectric and hydrodynamic models for formations with different fluid saturations in analysis of HFEL data.

To construct a closed hydrodynamic model of an invaded zone, it is necessary to take into account results of inversion of electromagnetic logging over the entire set of formations for the borehole in question and results of petrophysical studies of geological sections of neighboring boreholes.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 03-05-64210).

REFERENCES

1. *Development of Research into Filtration Theory in the* USSR [in Russian] Nauka, Moscow (1969).
2. B. T. Zhumagulov, N. V. Zubov, V. N. Monakhov, and Sh. S. Smagulov, *New Computer Technologies in Petroleum Production*, Ilym, Alma-Ata (1966).
3. N. A. Gukasov, *Handbook on Fluid Flow Mechanics and Hydrodynamics in Drilling* [in Russian] Nedra, Moscow (1982).
4. E. G. Leonov and V. I. Isaev, *Fluid Mechanics in Drilling* [in Russian], Nedra, Moscow (1987).
5. A. A. Samarskii, *Theory of Difference Schemes* [in Russian], Nauka, Moscow (1977).
6. A. A. Kashevarov, "Mathematical modeling of salt transport processes by coupled subsurface and surface water flows," *J. Appl. Mech. Tech. Phys.*, **39**, No. 4, 584–592 (1998).
7. J. H. Zhange, Q. Hu, and Z. H. Liu, "Estimation of true formation resistivity and water saturation with a time-lapse induction logging method," *Soc. Profes. Well Log Anal.*, **40**, No. 2, 138–148. (1999).
8. I. Yeltsov, M. Épov, A. Kashevarov, et al., "Applying inverse problems of geoelectrics and hydrodynamics for characterization of formation properties," in: *Abstracts of the 1st Int. Conf. on Inverse Problems: Modeling and Simulation* (Fethiye, Turkey, July 14–21, 2002), Kocaeli Univ., Fethiye (2002), pp. 175–176.
9. A. N. Petrov, K. N. Kayurov, M. I. Epov, et al., "New firmware nine-probe high-frequency electromagnetic logging system," in: *Electric and Electromagnetic Research Techniques in Oil–Gas Wells* [in Russian], Joint Institute of Geology, Geophysics, and Mineralogy, Siberian Division, Russian Academy of Sciences, Novosibirsk (1999), pp. 122–130.
10. I. N. El'tsov, M. I. Epov, V. N. Ul'yanov, et al. "Analysis and inversion of well logs in the MFS VIKIZ-98 system," *Karotazhnik*, **74,** 70–84 (2000).